THE UNIVERSITY OF MALTA
DEPARTMENT OF MATHEMATICS

MATHEMATICAL FORMULAE

UNIVERSITY PRESS, MSIDA, MALTA
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MMXXII

CONTENTS

MENSURATION

Circle

Area of a circle, radius r is πr^2
Circumference of circle is $2\pi r$

Sphere

Volume of a sphere, radius r, is $\frac{4}{3}\pi r^3$
Surface area of sphere is $4\pi r^2$

Right circular cylinder

Volume of cylinder, radius r and height h is $\pi r^2 h$
Curved surface area is $2\pi rh$

Right circular cone

Volume of cone, radius r, and height h is $\frac{1}{3}\pi r^2 h$
Curved surface area is πrl where l is the slant height of the cone.

ALGEBRA

Factors

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Permutations and Combinations

$$^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = \binom{\mathrm{n}}{\mathrm{r}} = \frac{\mathrm{n}!}{\mathrm{r}!(\mathrm{n}-\mathrm{r})!}$$

$$^{\mathrm{n}}\mathrm{P}_{\mathrm{r}} = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!}$$

Finite Series

$$\sum_{q=0}^{n-1}(a+qd) = \frac{n}{2}[2a+(n-1)d]; \quad \sum_{q=0}^{n-1} ar^q = \frac{a(1-r^n)}{1-r}$$

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1); \quad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1); \quad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}\,x^2 + \cdots + \binom{n}{r}x^r + \cdots + x^n \quad (n \text{ +ve int.})$$

de Moivre's Theorem

If n is an integer, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

If n is a rational number, $\cos n\theta + i\sin n\theta$ is one of the values of $(\cos\theta + i\sin\theta)^n$.

HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh^{-1} x = \ln[x + \surd(x^2 + 1)]$$

Principal value of $\cosh^{-1} x = \ln[x + \surd(x^2 - 1)]$ $\qquad (x \geq 1)$

$$\tanh^{-1} x = \tfrac{1}{2} \ln \left| \frac{1+x}{1-x} \right| \qquad (|x| < 1)$$

CIRCULAR FUNCTIONS

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\left.\begin{array}{ll} \text{If } \sin\theta = \sin\alpha, & \text{then } \theta = n\pi + (-1)^n\alpha \\ \text{If } \cos\theta = \cos\alpha, & \text{then } \theta = 2n\pi \pm \alpha \\ \text{If } \tan\theta = \tan\alpha & \text{then } \theta = n\pi + \alpha \end{array}\right\} \text{where } n = 0, \pm 1, \pm 2, \ldots$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$
$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$
$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$
$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$
$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$
$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\text{If } \tan\frac{A}{2} = t, \text{ then } \sin A = \frac{2t}{1 + t^2}; \quad \cos A = \frac{1 - t^2}{1 + t^2}$$

COORDINATE GEOMETRY

Perpendicular distance from (h,k) to $ax+by+c=0$ is $\left|\dfrac{ah+bk+c}{\sqrt{(a^2+b^2)}}\right|$

The acute angle between two lines with gradients m_1, m_2 is
$\tan^{-1}\left|\dfrac{\mathrm{m}_1-\mathrm{m}_2}{1+\mathrm{m}_1\mathrm{m}_2}\right|$

Area of Triangle is
$|\tfrac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]| = \tfrac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$

Circle
The equation $x^2+y^2+2gx+2fy+c=0$ represents a circle with centre at $(-g,-f)$ and radius $\sqrt{(g^2+f^2-c)}$.

The parametric equations of a circle with centre at (a,b) and radius r are $x=a+r\cos t$, $y=b+r\sin t$.

Point dividing P_1P_2 in the ratio $k:1$ has coordinates

$$\left(\frac{x_1+kx_2}{1+k}, \frac{y_1+ky_2}{1+k}, \frac{z_1+kz_2}{1+k}\right)$$

Angle ϕ between two lines with direction cosines l, m, n :
l', m', n' is given by $\cos\phi = \dfrac{\pm(ll'+mm'+nn')}{\sqrt{(l^2+m^2+n^2)}\sqrt{(l'^2+m'^2+n'^2)}}$

Distance from $P_1(x_1,y_1,z_1)$ to plane $Ax+By+Cz+D=0$ is

$$\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{(A^2+B^2+C^2)}}\right|$$

Plane distance p from origin, direction cosines of normal l, m, n,

$$lx+my+nz=p.$$

Line through (x_1,y_1,z_1), direction cosines l, m, n.

$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}=t.$$

CALCULUS

I. INFINITE SERIES

Taylor's Theorem

$$f(a+x) = f(a) + xf'(a) + \frac{x^2}{2!}f''(a) + \cdots + \frac{x^{r-1}}{(r-1)!}f^{(r-1)}(a) + \cdots,$$

with 'remainder term' $\frac{x^r}{r!}f^{(r)}(a+\theta x)$, where $0 < \theta < 1$.

Maclaurin's Theorem

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^{r-1}}{(r-1)!}f^{(r-1)}(0) + \cdots,$$

with 'remainder term' $\frac{x^r}{r!}f^{(r)}(\theta x)$, where $0 < \theta < 1$.

$$\exp x \equiv e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} + \cdots \qquad *$$

$$\log_e(1+x) \equiv \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{r-1}\frac{x^r}{r} + \cdots$$

valid for $-1 < x \leq 1$.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \cdots \qquad *$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^r \frac{x^{2r}}{(2r)!} + \cdots \qquad *$$

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2r+1}}{(2r+1)!} + \cdots \qquad *$$

$$\cosh x = \tfrac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2r}}{(2r)!} + \cdots \qquad *$$

* *These series are valid for all finite x.*

II DERIVATIVES

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
e^x	e^x
$a^x \ (a > o)$	$a^x \ln a$
$\log_e x \equiv \ln x$	$\frac{1}{x}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
uv	$uv' + u'v$
$\frac{u}{v}$	$(vu' - uv')/v^2$

III INTEGRALS (*Constants of integration are omitted;* $\ln a \equiv \log_e a$)

$f(x)$	$\int f(x)dx$
$\dfrac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{(a^2+x^2)}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{(a^2+x^2)}}$	$\ln\{x+\sqrt{(x^2+a^2)}\}$ or $\sinh^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{x}{\sqrt{(a^2+x^2)}}$	$\sqrt{(a^2+x^2)}$
$\dfrac{1}{\sqrt{(x^2-a^2)}}$	$\ln\{x+\sqrt{(x^2-a^2)}\}$ or $\cosh^{-1}\left(\dfrac{x}{a}\right)$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$\ln(\sec x)$
$\cot x$	$\ln(\sin x)$
$\sec x$	$\ln(\sec x+\tan x)$ or $\ln\left\{\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right\}$
$\operatorname{cosec} x$	$\ln\tan\dfrac{x}{2}$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$
$u\dfrac{dv}{dx}$	$uv-\int v\dfrac{du}{dx}\,dx$

IV APPLICATIONS

For a curve $y = f(x)$, $a \leq x \leq b$.

Area between curve and x-axis $= \int_a^b y \; dx$

$$\textit{Mean value} = \frac{1}{b-a} \int_a^b y \; dx$$

Volume of revolution about x-axis $= \pi \int_a^b y^2 \; \mathrm{dx}$

Centroid of area between curve and x-axis has coordinates

$$\bar{x} = \frac{\int_a^b xy \; dx}{\int_a^b y \; dx}; \qquad \bar{y} = \frac{\int_a^b {}^1\!/_2 y^2 \; dx}{\int_a^b y \; dx}$$

Centroid of solid of revolution about x-axis:

$$\bar{x} = \frac{\int_a^b xy^2 \; dx}{\int_a^b y^2 \; dx}$$

For the area shown in Figure 1

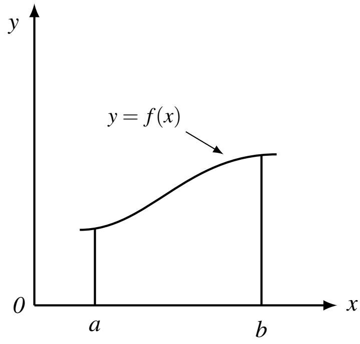


Fig. 1

First moment about x-axis $= \int_a^b \frac{1}{2}y^2\,dx$

First moment about y-axis $= \int_a^b xy\,dx$

Second moment about x-axis $= \int_a^b \frac{1}{3}y^3\,dx$

Second moment about y-axis $= \int_a^b x^2y\,dx$

For the solid of revolution shown in Figure 2

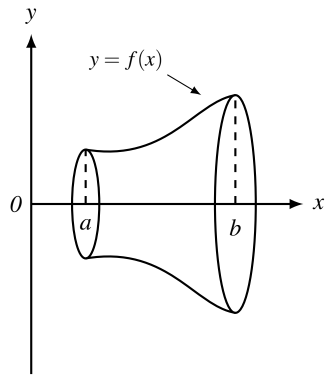


Fig. 2

First moment about xy-plane $= 0$

First moment about yz-plane $= \pi\int_a^b xy^2\,dx$

Second moment about x-axis $= \pi\int_a^b \frac{1}{2}y^4\,dx$

Second moment about y-axis $= \pi\int_a^b y^2\left(x^2+\frac{y^2}{4}\right)dx$

$$\text{Length of arc} = \int_a^b \sqrt{\{1 + \left(\frac{dy}{dx}\right)^2\}}\, dx = \int_{t_1}^{t_2} \sqrt{(\dot{x}^2 + \dot{y}^2)}\, dt$$

$$\text{Area of surface of revolution} = 2\pi \int_a^b y\sqrt{\{1 + \left(\frac{dy}{dx}\right)^2\}}\, dx$$

$$= 2\pi \int_{t_1}^{t_2} y\sqrt{(\dot{x}^2 + \dot{y}^2)}\, dt$$

$$\text{Radius of curvature } \rho = \frac{\{1 + \left(\frac{dy}{dx}\right)^2\}^{3/2}}{\frac{d^2y}{dx^2}} = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{\dot{x}\ddot{y} - \ddot{x}\dot{y}}$$

Polar Coordinates

$$\text{Area enclosed by curve} = {}^1\!/_2 \int_{\theta_1}^{\theta_2} r^2\, d\theta$$

$$\text{Length of arc} = \int_{\theta_1}^{\theta_2} \sqrt{\{r^2 + \left(\frac{dr}{d\theta}\right)^2\}}\, d\theta = \int_{r_1}^{r_2} \sqrt{\{1 + r^2\left(\frac{d\theta}{dr}\right)^2\}}\, dr$$

$$\text{Radius of curvature } \rho = r \Big/ \frac{dp}{dr}$$

V APPROXIMATIONS

Trapezoidal Rule:

$$\int_a^b y\,dx \approx \tfrac{1}{2}h\{(y_o + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\}$$

Simpson's rule (n even)

$$\int_a^b y\,dx \approx \tfrac{1}{3}h\{(y_o + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1})$$
$$+ 2(y_2 + y_4 + \cdots + y_{n-2})$$

Newton's approximation to a root of $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

VECTORS

Line through point, position vector **a**, parallel to **b**

$$r = \mathbf{a} + t\mathbf{b}$$

Position vector of a point dividing the line joining P, Q with position vectors **p**, **q** in the ratio $\lambda : \mu$ is $\dfrac{\lambda\mathbf{p} + \mu\mathbf{q}}{\lambda + \mu}$

Plane through point, position vector **a**, perpendicular to **n**

$$(\mathbf{r} - \mathbf{a}) \,.\, \mathbf{n} = \mathbf{0}$$

Scalar product $= \mathbf{a}_1 \,.\, \mathbf{a}_2 = a_1 a_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

Vector product $= \mathbf{a}_1 \times \mathbf{a}_2 = \mathbf{a}_1 \wedge \mathbf{a}_2 = a_1 a_2 \sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

MECHANICS

Centre of mass

Arc, radius r, angle 2θ	$r\sin\theta/\theta$ from centre
Sector of circle, radius r, angle 2θ	$\frac{2}{3}r\sin\theta/\theta$ from centre
Hemisphere, radius r	$\frac{3}{8}r$ from centre
Hemispherical shell, radius r	$\frac{1}{2}r$ from centre
Solid cone, height h	$\frac{1}{2}$ from vertex
Conical shell, height h	$\frac{2}{3}h$ from vertex

Moments of inertia

Rod, length $2l$, about perpendicular axis through centre	$\frac{1}{3}ml^2$
Disc, radius r, about perpendicular axis through centre	$\frac{1}{2}mr^2$
Hoop, radius r, about diameter	$\frac{1}{2}mr^2$
Solid sphere, radius r, about diameter	$\frac{2}{5}mr^2$
Spherical shell, radius r, about a diameter	$\frac{2}{3}mr^2$
Parallel axes theorem	$I_A = I_G + M(GA)^2$
Perpendicular axes theorem for a lamina	$I_{oz} = I_{ox} + I_{oy}$

Simple harmonic motion

$$\frac{d^2y}{dt^2} = -\omega^2 x,\ \left(\frac{dx}{dt}\right)^2 = \omega^2(a^2 - x^2),\ x = a\sin(\omega t + \varepsilon)$$

Compound pendulum

Period $= 2\pi\sqrt{(k^2 + h^2)/gh}$

Components of acceleration

$\ddot{r} - r\dot{\theta}^2$ along radius vector

$2\dot{r}\dot{\theta} + r\ddot{\theta}$ perpendicular to radius vector

PROBABILITY

Probability laws

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A) \times P(B \mid A)$$

Discrete variable X with probability function $P(X = x)$	Continuous variable X with probability density function $f(X)$
Distribution function $F(X)$ $F(x_o) = P(X \leq x_o)$ $= \sum_{x \leq x_o} P(x)$	$F(x) = P(X < x)$ $= \int_{-\infty}^{x_o} f(x)\,dx$
Expectation of X $E(X) = \Sigma x P(X = x)$	$E(X) = \int x f(x)\,dx$
Expectation of $g(x)$ $E[g(x)] = \Sigma g(x) P(X = x)$	$E[g(X)] = \int g(x) f(x) dx$
Variance σ^2 $\mathrm{Var}(X) = E[\{X - E(X)^2\}]$	
Covariance $\mathrm{Cov}(X_1, X_2) = E[\{X_1 - E(X_1)\}\{X_2 - E(X_2)\}]$	
Correlation coefficient $\rho_{12}(X_1, X_2)$ $\rho_{12} = \dfrac{\mathrm{Cov}(X_1, X_2)}{\sqrt{\{\mathrm{Var}(X_1)\,\mathrm{Var}(X_2)\}}}$	
Linear regression coefficient, β_{12}, for X_1 on X_2 $\beta_{12} = \dfrac{\mathrm{Cov}(X_1, X_2)}{\mathrm{Var}(X_2)}$	

Probability generating function $G(z)$

$$G(z) = P(0) + P(1)z + P(2)z^2 + \cdots + P(r)z^r + \cdots,$$

where $P(r) = P(X = r)$

Binomial distribution (X, p, N)

$$P(X = k) = \binom{N}{k} p^k (1-p)^{N-k}$$

$$E(X) = Np$$

$$\text{Var}(X) = Np(1-p)$$

$$G(z) = [pz + (1-p)]^N$$

Poisson distribution (X, m)

$$P(X = k) = \frac{e^{-m} m^k}{k!}$$

$$E(X) = m$$

$$\text{Var}(X) = m$$

$$G(z) = e^{-m} e^{mz}$$

Normal distribution

If X is distributed $N(\mu, \sigma^2)$ then $\dfrac{X-\mu}{\sigma}$ is distributed $N(0,1)$

where σ is the standard deviation and σ^2 is the variance.

STATISTICS

μ, σ^2 population mean and variance

X_i ith random selection in a sample size n

Sample mean

$$\overline{X} = \frac{1}{n} \Sigma X_i$$

Sample variance

$$S^2 = \frac{1}{n-1} \Sigma (X_i - \overline{X})^2$$

$$E(S^2) = \sigma^2$$

$$E(\overline{X}) = \mu$$

$$\text{Var}(X) = \frac{\text{Var}(\overline{X})}{n} = \frac{\sigma^2}{n}$$

One sample t-test

$$t_{n-1} = \frac{\overline{X} - \mu_o}{S/\sqrt{n}}$$

Two sample t-test

$$t_{n_1+n_2-2} = \frac{\overline{X}_1 - \overline{X}_2}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\text{where } S^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

and n_1, n_2 are the sizes of the two samples

Paired sample t-test

$$t_{n-1} = \frac{\overline{Y}}{S\sqrt{\left(\frac{1}{n}\right)}}$$ where $Y_j = X_{1j} - X_{2j}$ $(j = 1, 2, 3, \ldots, n)$ and

$s^2 = \text{Var(y)}$

Spearman's rank correlation coefficient ρ

$$\rho = 1 - \frac{6\Sigma\, d^2}{n(n^2-1)}$$

Kendall's rank correlation coefficient r

$$r = \frac{\begin{pmatrix}\text{Number of agreed} \\ \text{pair rankings}\end{pmatrix} - \begin{pmatrix}\text{Number of different} \\ \text{pair rankings}\end{pmatrix}}{\text{Number of pairs}} = \frac{S}{\frac{1}{2}n(n-1)}$$

Paired sample Wilcoxon $(n > 8)$

$T =$ (sum of the ranks with the less frequent sign)

$Z = \frac{T - \overline{T}}{s}$ distributed $N(0,1)$; $\overline{T} = \frac{n(n+1)}{4}$; $s^2 = \frac{n(n+1)(2n+1)}{24}$

Table 1 The standardised normal distribution

Entry represents area under the standardized normal distribution from the mean to Z

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3079	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4430	.4441
1.6	.4452	.4463	.4474	.4485	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4700	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4762	.4767
2.0	.4773	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4865	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4980	.4980	.4981
2.9	.4981	.4982	.4983	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.49865	.49869	.49874	.49878	.49882	.49886	.49889	.49893	.49896	.49900
3.1	.49903	.49906	.49910	.49913	.49916	.49918	.49921	.49924	.49926	.49929
3.2	.49931	.49934	.49936	.49938	.49940	.49942	.49944	.49946	.49948	.49950
3.3	.49952	.49953	.49955	.49957	.49958	.49960	.49961	.49962	.49964	.49965
3.4	.49966	.49968	.49969	.49970	.49971	.49972	.49973	.49974	.49975	.49976
3.5	.49977	.49978	.49978	.49979	.49980	.49981	.49981	.49982	.49983	.49983
3.6	.49984	.49985	.49985	.49986	.49986	.49987	.49987	.49988	.49988	.49989
3.7	.49989	.49990	.49990	.49990	.49991	.49991	.49992	.49992	.49992	.49992
3.8	.49993	.49993	.49993	.49994	.49994	.49994	.49994	.49995	.49995	.49995
3.9	.49995	.49995	.49996	.49996	.49996	.49996	.49996	.49996	.49997	.49997

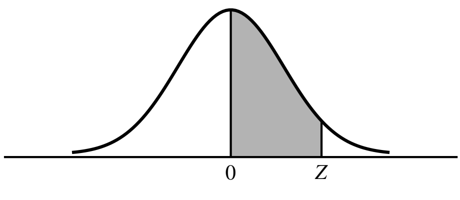


Table 2 Percentage points of Student's t-distribution

α	.10	.05	.025	.01	.005	.001
v						
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
60	1.296	1.671	2.000	2.390	2.660	3.232
120	1.289	1.658	1.980	2.358	2.617	3.160
∞	1.282	1.645	1.960	2.326	2.576	3.090

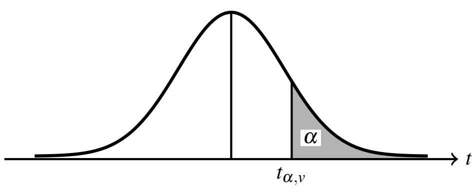


Table 3 Percentage points of the χ^2 distribution

α	.995	.99	.975	.95	.50	.20	.10	.05	.025	.01	.005
v											
1	0.000	0.0002	0.0010	0.0039	0.45	1.64	2.71	3.84	5.02	6.63	7.88
2	0.010	0.020	0.051	0.103	1.39	3.22	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	2.37	4.64	6.25	7.81	9.35	11.34	12.84
4	0.207	0.30	0.484	0.71	3.36	5.99	7.78	9.49	11.14	13.28	14.86
5	0.412	0.55	0.831	1.15	4.35	7.29	9.24	11.07	12.83	15.09	16.75
6	0.676	0.87	1.24	1.64	5.35	8.56	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	6.35	9.80	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	7.34	11.03	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	8.34	12.24	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	9.34	13.44	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	10.34	14.63	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	11.34	15.81	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	12.34	16.99	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	13.34	18.15	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	14.34	19.31	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	15.34	20.47	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	16.34	21.62	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	17.34	22.76	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	18.34	23.90	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	19.34	25.04	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	20.34	26.17	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	21.34	27.30	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	22.34	28.43	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	23.34	29.55	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	24.34	30.68	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	25.34	31.80	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	26.34	32.91	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	27.34	34.03	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	28.34	35.14	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	29.34	36.25	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	39.34	47.27	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	49.33	58.16	63.17	67.51	71.41	76.15	79.49
60	35.53	37.48	40.48	43.19	59.33	68.97	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	69.33	79.71	85.53	90.53	95.02	100.43	104.2
80	51.17	53.34	57.15	60.39	79.33	90.41	96.58	101.88	106.63	112.33	116.3
90	59.20	61.75	65.85	69.13	89.33	101.05	107.57	113.15	118.14	124.12	128.3
100	67.33	70.06	74.22	77.93	99.33	111.67	118.50	124.34	129.56	135.81	140.2

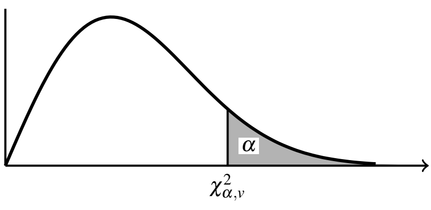
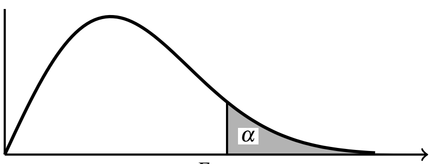


Table 4 Upper percentage points of the F-distribution

(a) $\alpha = 0.01$

v_1 / v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
1	4052.2	4999.5	5403.4	5624.6	5763.7	5859.0	5928.4	5981.1	6022.5	6055.8	6106.3	6157.3	6208.7	6234.6	6260.6
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.51
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38
6	13.75	10.93	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.67	3.59	3.51
14	8.86	6.52	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.90	3.81	3.67	3.52	3.37	3.29	3.21
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00
18	8.29	6.01	5.09	4.58	4.25	4.02	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.93	2.84
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67
23	7.88	5.66	4.77	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.59	2.50
27	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44
29	7.60	5.42	4.54	4.05	3.73	3.50	3.33	3.20	3.09	3.01	2.87	2.73	2.57	2.50	2.41
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.67	2.52	2.37	2.29	2.20
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.04	1.95	1.86
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.19	2.04	1.88	1.79	1.70



F_{α,v_1,v_2}

Table 4 (continued)

(a) $\alpha = 0.025$

v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
v_2															
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87	993.10	997.25	1001.4
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.47
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.45
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.95	1.82	1.76	1.69
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57

Table 4 (continued)

(a) $\alpha = 0.05$

v_1 / v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.10
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46